A Polarimeter to Measure the Complete State of Polarization of Scattered Solar Radiation

Ein Polarimeter zur Messung des vollständigen Polarisationszustandes gestreuter solarer Strahlung

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Received: May 17, 1974

Summary: This paper describes a new polarimeter for the measurement of the four Stokes-parameters of scattered solar radiation. Its wavelength range of sensitivity is 600–1000 nm. The four Stokes-parameters are obtained by simultanious measurements of 6 radiances. The maximum number of measurements is 166/sec.

The theory of measurements and calibrations has been discussed. At last, there is described one measurement of skylight of a cloudless but turbid atmosphere.

Zusammenfassung: Es wird ein neuartiges Polarimeter zur Messung der vier Stokes-Parameter gestreuter solarer Strahlung – im Wellenlängenbereich von 600–1000 nm – beschrieben. Die vier Stokes-Parameter werden aus der simultanen Messung von sechs Strahldichten berechnet. Die Meßzeit hierfür beträgt $6 \cdot 10^{-3}$ sec.

Die Meß- und Eichtheorie wird diskutiert und zuletzt eine Messung bei wolkenlosem Himmel beschrieben.

Résumé: On décrit un nouveau polarimètre pour mesurer les quatre paramètres de Stokes du rayonnement solaire diffus, dans le domaine spectral de 600 à 1000 nm. Les quatre paramètres de Stokes sont obtenus à partir de la mesure simultanée de six luminances énergétiques. Le temps de mesure est de 6.10^{-3} sec.

On discute la théorie de la mesure et de l'étalonnage et enfin, on décrit une mesure par ciel serein.

1. Introduction

Photoelectric polarimeters have been used in atmospheric photometry for a number of years (e. g. SEKERA 1957). Most of these polarimeters were only capable to measure the degree of polarization and sometimes its direction (e. g. COFFEEN 1972).

Theoretical and experimental investigations (EIDEN 1966) showed the importance of all four Stokes-parameters concerning scattering processes in polluted air. Another point is the speed of measurement. The speed must be high, when moveable objects (coulds etc.) or a large part of sky during the same elevation of sun are to be measured. All of this gave the starting point to develop a novel polarimeter about three years ago.

An excellent description of the state of polarization of light was introduced by G. G. STOKES (1852), the so called Stokes-parameters S_0, S_1, S_2, S_3. These have the dimension of radiance and may be defined in the following way for a finite spectral interval between λ and $\lambda + \Delta\lambda$.

$$\begin{aligned} S_0 &= L(0°, 0) + L(90°, 0) \\ S_1 &= L(0°, 0) - L(90°, 0) \\ S_2 &= L(45°, 0) - L(135°, 0) \\ S_3 &= B[L(45°, R) - L(135°, R)] - C\,S_2 \end{aligned} \tag{1}$$

$$B = \frac{1}{\sum_\lambda E_\lambda \Delta\lambda} \sum_\lambda \frac{E_\lambda \Delta\lambda}{\sin \tau_\lambda} \tag{2}$$

$$C = \frac{1}{\sum_\lambda E_\lambda \Delta\lambda} \sum_\lambda \frac{E_\lambda \Delta\lambda}{\sin \tau_\lambda} \cos \tau_\lambda \tag{3}$$

The first argument of the radiance L gives the direction of polarization and the second the retardation R. If the retardation is $\lambda/4$ and the spectral interval $\Delta\lambda$ is extremely small, then are $B \simeq 1$ and $C \simeq 0$. In the other cases B and C must be computed with respect to the spectral sensitivity E of the instrument and the phase-difference $\tau_\lambda = 2\pi R/\lambda$. The four Stokes-parameters may be interpreted as the four components of a mathematical vector $\vec{S}$.

Fig. 1 gives a survey of the last three Stokes-parameters and its grafical analogon.

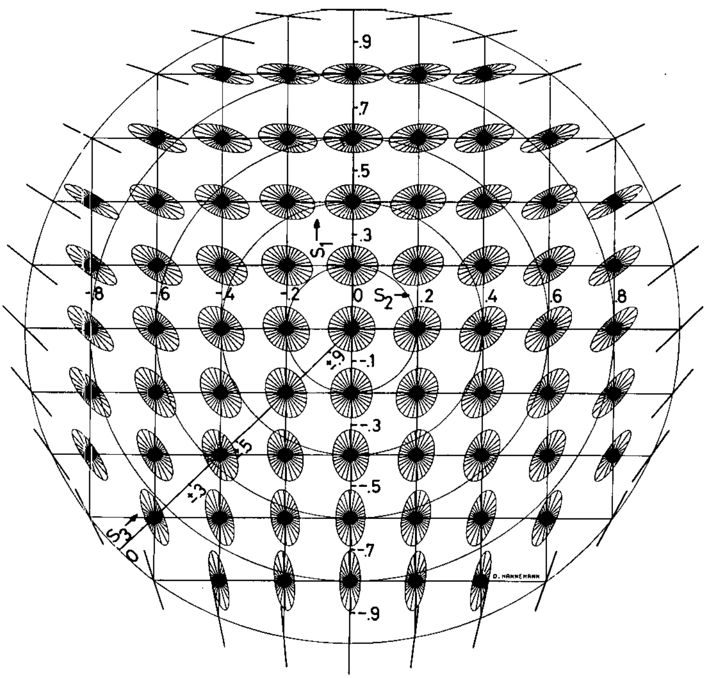


Fig. 1. The states of polarization as the relations of the three Stokes-parameters

Bild 1. Polarisationszustand als Funktion der drei Stokes-Parameter

The degree of polarization is defined as

$$P = \frac{(S_1^2 + S_2^2 + S_3^2)^{1/2}}{S_0}, \quad (0 \leqslant P \leqslant 1). \tag{4}$$

The direction of polarization φ and the ellipticity e depend on the Stokes-parameters in the following way

$$\varphi = \frac{1}{2} \arctan \frac{S_2}{S_1}, \quad (0^\circ \leqslant \varphi \leqslant 180^\circ) \tag{5}$$

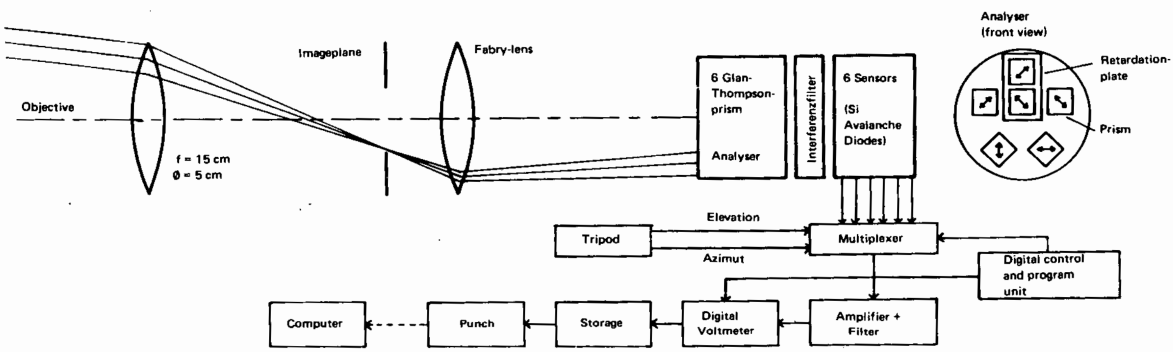
$$e = \tan \left[\frac{1}{2} \left(\arcsin \frac{S_3}{S_0\, P} \right) \right], \quad (-1 \leqslant e \leqslant 1) \tag{6}$$

$e \lesseqgtr 0$, right-or left-handed.

The design of the polarimeter described here is based on the definitions in Eq. 1–3 and on the requirement to obtain quick observations. It is capable of measuring the four Stokes-parameters of skylight at one point during $6 \cdot 10^{-3}$ seconds. The possible wavelength range of observation is 600–1000 nm.

2. The Polarimeter

The process of measurement is based on Eqs. (1). At the back of a normal photometric optical system are six analysers with different qualities, a filter and six sensors (see Fig. 2).



- **Fig. 2.** Schematic representation of the polarimeter
- **Bild 2.** Schematischer Aufbau des Polarimeters

The light of any point of the object illuminates all of the six analyser channels. The analyser channels detect the radiance with the following qualities (see Eq. (1)):

$$L(0^\circ, 0),\ L(45^\circ, 0),\ L(90^\circ, 0),\ L(135^\circ, 0),\ L(45^\circ, R),\ L(135^\circ, R).$$

A multiplexer connects the output of all six channels in a frequency up to 1 kHz, with a digital voltmeter. Then an electronic storage takes up to fifty data and transmits them to a tape punch recorder.

The four Stokes-parameters could easily be computed from the output of all six channels, if all of the channels had the same sensitivity. In general this is not possible because the characteristics of these channels (i. e. detectors and analysers) are too different. In the next chapter this problem will be solved. Another difficulty is the effect of the optical devices concerning the incident Stokes-vector; it will be managed in the next chapter too.

3. Calibrations

3.1. Sensor Characteristics

First it is necessary to determine the sensor sensitivity V in dependence of the incident radiance L. The sensors used here were Si-avalanche diodes. They have not a linear response which results essentially from the built up circuit.

Using a calibration lamp and a set of calibrated neutral filters it is possible to determine the characteristics $U_k = f_k(L)$, k = 1 . . 6 = channel number, U = sensor voltage. Additional one of the voltages (without neutral filter) will be used as reference value to compute the general momentary sensitivity of the polarimeter. After the sensor characteristics are determined it is necessary to accomplish a single calibration measurement without neutral filter using the calibration lamp before each measurement. Using channel 6, the momentary sensitivity is

$$V_{rel} = U_6\,(\text{momentary})/U_6\,(\text{first calibration}).$$

All of the six channels have slightly different characteristics. As an example one of these is shown in Fig. 3.

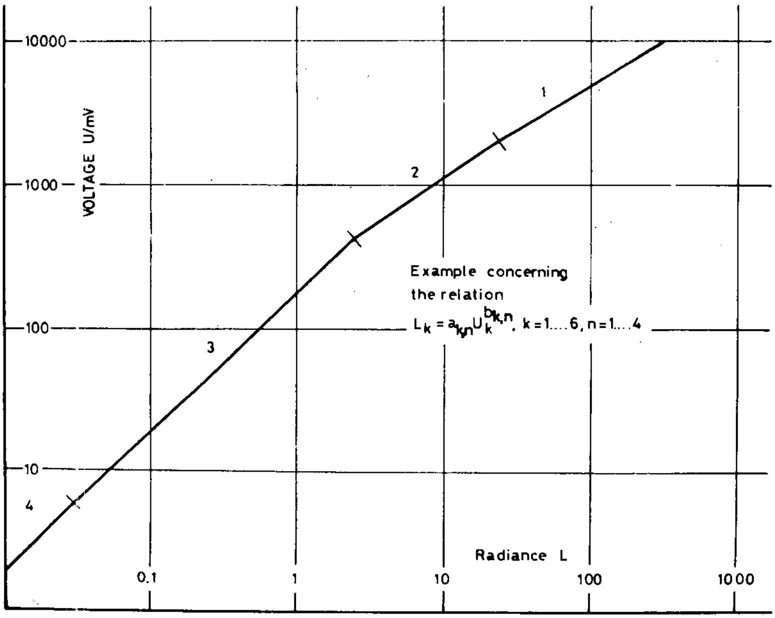


Fig. 3. Typical characteristics of one of the Si-avalanche diodes

Bild 3. Typische Charakteristik einer Si-Avalanche Diode

Its slope, if plotted in a double logarithmic way shows four linear parts. Hence, we can write

$$L_k = a_{n,k}\, U_k^{b_{n,k}}, \; n = 1 \ldots . 4. \tag{7}$$

3.2. Channel Sensitivity

The use of 6 different channels arises the difficulty to determine the relative channel sensitivity concerning one of them (here arbitrary channel 6). It is impossible to measure the relative channel sensitivity directly because the optical devices polarize the incident unpolarized light. To solve this problem we can utilize the fact of overdetermination in the Eqs. (1). In principle it is necessary to make only four measurements to determine the four Stokes-parameters. Thus, we can write

$$\begin{aligned} S_0^{(1)} &= L(0°, 0) + L(90°, 0) = L_1 + L_2 \\ S_0^{(2)} &= L(45°, 0) + L(135°, 0) = L_3 + L_4 \\ S_0^{(3)} &= L(45°, R) + L(135°, R) = L_5 + L_6 \,. \end{aligned} \tag{8}$$

Evidently the three first Stokes-parameters $S_0^{(1..3)}$ must be equal and therefore it is correct to write

$$\begin{aligned} L_1 + L_2 - L_5 - L_6 &= 0 \\ L_3 + L_4 - L_5 - L_6 &= 0. \end{aligned} \tag{9}$$

To apply Eq. (7) after a normal measurement we must write

$$L_k = a_{n,k}\,(U_k/V_k)^{b_{n,k}}, \tag{10}$$

with

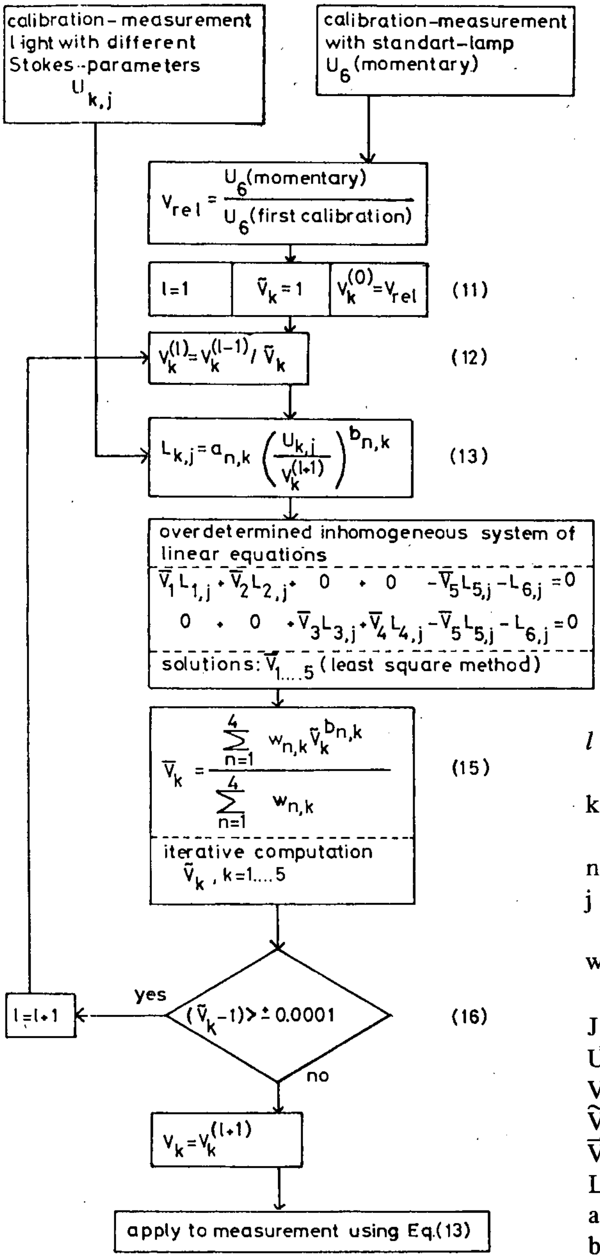
$$\begin{aligned} V_k &= V_{rel}/V'_k, \; k = 1 \ldots 6, \\ V_k &= \text{sensitivity factor}, \\ V_6 &= 1, \; V'_k = \text{relative channel sensitivity.} \end{aligned}$$

Using calibration measurements which are described in chapter 3.3. it is possible to built up a system of linear equations with respect to Eq. (9) and in addition to that one may add five unknown factors $\overline{V}_k$, k = 1 . . 5 (see Fig. 4, Eq. (14)).

Fig. 4 gives a summary of the complete computational method. The mean sensitivities $\overline{V}_k$ which are computed by Eq. (14) are defined by Eq. (15). This computational system must be repeated about 4 to 5 times because some sensor voltages will change from one part into another part of sensor characteristics after using the computed sensitivity factor.

3.3. Calibration – Matrix

The effect of optical components on the light beam is represented by a linear transformation of the Stokes-vector $\vec{S}$. The transformation matrix (often called Mueller matrix) is a 4 × 4 matrix, because the Stokes-vector is a 4-vector (see e. g. SHURCLIFF 1962).



l = number of iteration
= 1 . . . 6

k = channel number
= 1 . . . 6

n = part of sensor characteristic

j = calibration-measurement-number
= 1 . . . J

$w_{n,k}$ = quantity of $L_{n,k}$ in the n-part sensor characteristics from channel k.

J = quantity of calibration measurements

U = voltage of sensors

V_k = sensitivity-factor of channels

$\tilde{V}_k$ = sensitivity-factor of channels for the iterations

$\bar{V}_k$ = mean sensitivity-factor defined by Eq. (15)

$L_{k,j}$ = radiance

$a_{n,k}$ =, $b_{n,k}$ = } constants of sensor characteristics

- **Fig. 4.** Scheme of channel sensitivity computation
- **Bild 4.** Schema zur Berechnung der Kanal-Empfindlichkeiten

An optical system like the polarimeter can be described by a 4 × 4 matrix, too, but in general it is impossible to compute theoretically the matrix. We must compute the matrix (here called calibration-matrix) using light with at least four defined but different Stokes-vectors which incidents upon the polarimeter. The easiest way to compute the calibration-matrix is to use the following calibration Stokes-vectors:

(1, 0, 0, 0) (1, 1, 0, 0), (1, 0, 1, 0), (1, 0, 0, 1).

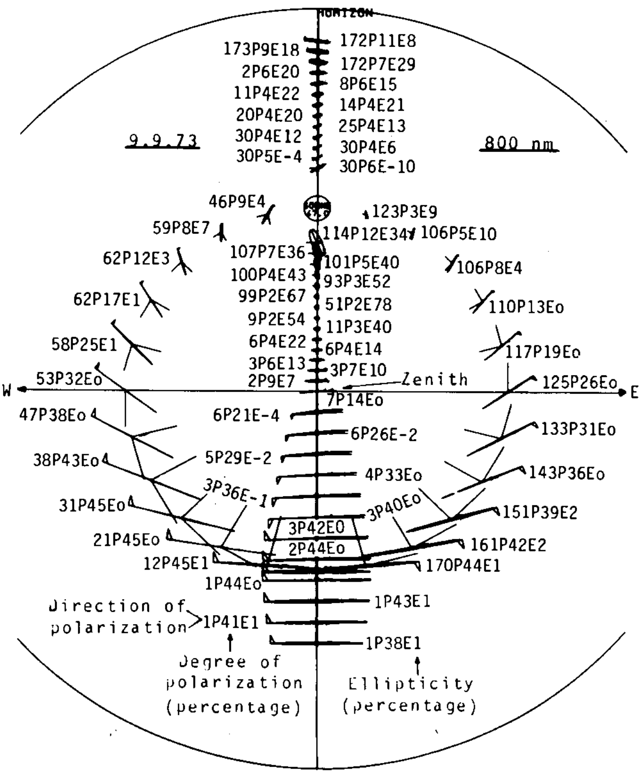
The first represents unpolarized light, but it is extremely difficult to procedure it. Also the inaccuracies to making and measuring light with special Stokes-vectors are another problem.

Therefore it is better to use more than four different light sorts. The last three calibration Stokes-vectors are the basis vectors of the Poincaré-sphere (see e. g. SHURCLIFF 1962). Additional to these basis vectors it is convenient to take vectors which fill the Poincaré-sphere symmetrically.

At first we take the negative basis vectors, then the eight vectors which are placed symmetrically between the six positive and negative basis vectors. In this way we get fourteen calibration Stokes-vectors.

Knowing these calibration vectors and measuring the corresponding voltages at the six analyser channels it is possible to make a complete calibration of the channel sensitivities (chapter 3.2.) using the calibration measurements. The next step is to compute the radiances and then the measured Stokes-vektors – that are the calibration vectors and the vectors of the real measurement. After that we can compute the sixteen elements of the calibration matrix. The computation is carried out in an overdetermined system of linear equations using the incident and measured calibration vectors. At last the calibration matrix must be inversed and then multiplicated with the measured Stokes-vectors.

Then the measured Stokes-vectors are reduced to the incident ones.



● **Fig. 5**
Measurements of the polarisation of downward scattered sun radiation during a cloudless day at Hohenpeißenberg, Bavaria.

● **Bild 5**
Polarisationsmessungen an der abwärts gestreuten Sonnenstrahlung in der Atmosphäre über Hohenpeißenberg, Bayern, während eines wolkenlosen Tages.

4. Accuracy

Normally we repeat a measurement six times, and then there still remains a statistical error of the mean voltage from 0.1–0.02 %.

A good way to check the total accuracy of the polarimeter is to illuminate it with different light sorts with well-known Stokes-parameters. A comparision between the measured and the incident Stokes-vectors shows the total accuracy of the polarimeter.

This was done and the following inaccuracies were found:

Ellipticity:	$e \pm 0.01$
Direction of polarization:	$\varphi \pm 0.5^\circ$
Degree of polarization:	$P \pm 1\,\%$
Radiance (relatively):	$L \pm 1\,\%$

5. Example for Measurements

The measurement shown in Fig. 5 were made at the Observatorium Hohenpeißenberg, W. Germany, when the sky was cloudless, but slightly turbid. The length of the major axis of the ellipsis in Fig. 5 are proportional to the degree of polarization. The arrow on the ellipsis indicates the right – or left-handed circulation.

In near future there will follow another publications containing more measurements and its interpretations.

6. Outlook

The next step to take is to make measurements at different places during different atmospheric conditions. We want to prove the theoretical investigations about turbid atmospheres including the ellipticity.

Acknowledgements

This work has been supported by the Deutsche Forschungsgemeinschaft. The author thanks all those who have collaborated in the construction of the polarimeter. Discussions with R. H. GIESE, E. RASCHKE, D. HABLICK have been very enlightening.

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Equations

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$$P = \frac{(S_1^2 + S_2^2 + S_3^2)^{1/2}}{S_0}, \ (0 \leqslant P \leqslant 1) \tag{4}$$

$$\varphi = \tfrac{1}{2} \arctan \frac{S_2}{S_1}, \ (0° \leqslant \varphi \leqslant 180°) \tag{5}$$

$$e = \tan \{\tfrac{1}{2} (\arcsin \frac{S_3}{S_0 \cdot P}), \ (-1 \leqslant e \leqslant 1) \tag{6}$$

$e \lesseqgtr 0$, right- or left-handed

$$L_k = a_{n,k}\, U_k^{b_{n,k}}, \ n = 1 \ldots 4 \tag{7}$$

$$\begin{aligned} S_0^{(1)} &= L(0°, 0) + L(90°, 0) = L_1 + L_2 \\ S_0^{(2)} &= L(45°, 0) + L(135°, 0) = L_3 + L_4 \\ S_0^{(3)} &= L(45°, R) + L(135°, R) = L_5 + L_6 \end{aligned} \tag{8}$$

$$\begin{aligned} L_1 + L_2 - L_5 - L_6 &= 0 \\ L_3 + L_4 - L_5 - L_6 &= 0 \end{aligned} \tag{9}$$

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